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A model of bipedal locomotion on compliant legs

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SUMMARY

Simple mathematical models capable of walking or running are used to compare the merits of bipedal gaits. Stride length, duty factor (the fraction of the stride, for which the foot is on the ground) and the pattern of force on the ground are varied, and the optimum gait is deemed to be the one that minimizes the positive work that the muscles must perform, per unit distance travelled.

Even the simplest model, whose legs have neither mass nor elastic compliance, predicts the changes of duty factor and force pattern that people make as they increase their speed of walking. It predicts a sudden change to running at a critical speed, but this is much faster than the speed at which people make the change. When elastic compliance is incorporated in the model, unnaturally fast walking becomes uncompetitive. However, a slow run with very brief foot contact becomes the optimum gait at low speeds, at which people would walk, unless severe energy dissipation occurs in the compliance. A model whose legs have mass as well as elastic compliance predicts well the relationship between speed and stride length in human walking.

1. INTRODUCTION

People walk to travel slowly and run to go faster. In both gaits the legs move alternately, half a cycle out of phase with each other, but the two gaits are never the less very different. In walking, there are times when both feet are on the ground simultaneously because the duty factor (the fraction of the duration of the stride for which each foot is on the ground) is greater than 0.5. In running, however, the duty factor is less than 0.5 so there are times when both feet are off the ground. Force plate records of walking show that while each foot is on the ground it exerts two broad peaks of force with an intervening minimum (figure 1*a–c*). Records of running however show only one maximum, if we ignore the very brief initial peak of force due to the impact of the moving foot on the ground (figure 1*d*).

Our manner of movement also changes with speed, within each gait. Both in walking and in running, stride length increases with speed. (Stride length is the distance between corresponding points on successive footprints of the same foot.) As walking speed increases, the minimum between the two force maxima becomes deeper (figure 1*a–c*).

There is some evidence from measurements of oxygen consumption that the effect of these changes is to minimize the energy cost of locomotion, at each particular speed. The change from walking to running is made at approximately the speed at which walking and running have equal energy costs: below that speed walking is more economical than running and above it the reverse is true (Margaria 1976). The stride length that is normally used at any given walking speed is the one that minimizes energy consumption (Zarrugh 1974).

The relationship between the mechanical performance of muscles in locomotion and the rates at which they use metabolic energy is very imperfectly understood (Alexander 1991), but we can expect to find as a general rule that gaits that require less work from the muscles also consume less metabolic energy.

Alexander (1980) adopted the hypothesis that gaits are adapted to minimize the work required for locomotion, at each speed. A simple mathematical model was made to walk or run, using different duty factors and exerting different patterns of force on the ground. The work done by the legs in the course of each stride was calculated and the gait that minimized this work, for given speed and stride length, was assumed to be optimal. The model successfully predicted the changes in the patterns of force (with deeper or shallower minima) that accompany changes of human walking speed. It also predicted correctly that the change from walking to running should be made abruptly at a critical speed. Never the less, it had serious shortcomings.

1. It predicted too high a speed, for the transition from walking to running. It was suggested that this was due to its ignoring the elastic compliance of the leg and foot (on which see Ker *et al.* 1987).

2. It predicted infinitesimal duty factors (therefore, infinite ground forces) for optimal running gaits. This fault could be eliminated by setting an upper limit to the force that the legs can withstand, but it will be shown in this paper that it disappears in any case, when account is taken of leg compliance.

3. It ignored the masses of the legs and so the energy cost of swinging them forward for successive steps. Consequently, it predicted quite modest energy costs for walking with duty factors close to 1.0 (which would involve unrealistically fast forward swinging of the legs).

4. It assumed an empirical relationship between

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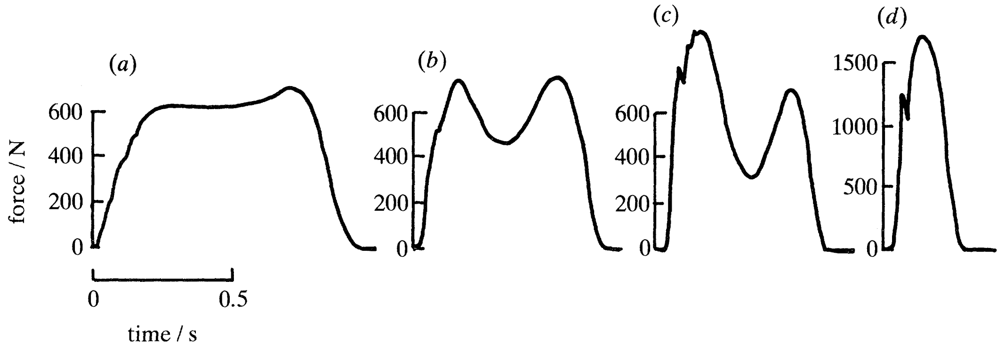

Figure 1. Records of the vertical component of the force exerted on the ground by one foot of a man walking at (*a*), 0.9 m s^{-1}; (*b*), 1.5 m s^{-1}; and (*c*), 2.1 m s^{-1}; and running at (*d*), 3.6 m s^{-1}. From the data of Alexander & Jayes (1980).

stride length and speed. If instead it had been used to calculate costs for different stride lengths, it would have shown infinitesimally short strides as optimal.

In this paper, I attempt to correct these faults by taking account of the mass and elastic compliance of the legs. I seek the optimal combination of duty factor, force pattern and stride length for each speed. I also improve the model by using simpler mathematical notation. I consider only bipedal gaits, but the model could easily be extended, as the previous one was (Alexander 1980), to symmetrical quadrupedal gaits.

Although the model has been made rather more realistic by taking account of leg mass and compliance, it remains very simple. My aim is not to imitate the movements of the human body but to show in the simplest possible way how they can be explained.

There are of course other simple models of bipedal walking (Mochon & McMahon 1980; McGeer 1990*a*) and running (Blickhan 1989; McMahon & Cheng 1990; McGeer 1990*b*). These have contributed greatly to our understanding of human movement. The distinctive features of the model presented in this paper (and of its predecessor, Alexander (1980)) are: first, that the same model can be made either to walk or to run, helping us to understand the gait transition; and secondly, that it seeks to explain the marked change in the pattern of ground forces that occurs as walking speed increases.

2. THE MODEL

The model (figure 2*a*) is two-dimensional, and consists of a rigid trunk and two legs. Each leg has a telescopic actuator that can exert force and make it lengthen and shorten, representing the muscles that change the length of real legs by flexing and extending the knee and ankle joints. A compression spring aligned with the long axis of the leg represents the elastic compliance conferred on real legs by the properties of tendons and ligaments (Ker *et al.* 1987). There is also a torque actuator at the hip, representing the muscles that flex and extend real hip joints. The total mass m of the model is made up of masses km for each of the legs and $(1-2k)m$ for the trunk. The trunk has its centre of mass located at the hip joints, and its moment of inertia is so large that pitching movements of the trunk are negligible. The legs are treated as point masses, located at a constant distance r from the hip joint: notice that this distance does not change as the leg lengthens and shortens.

The model walks or runs, setting down the feet at equal intervals. Each foot is alternately on the ground

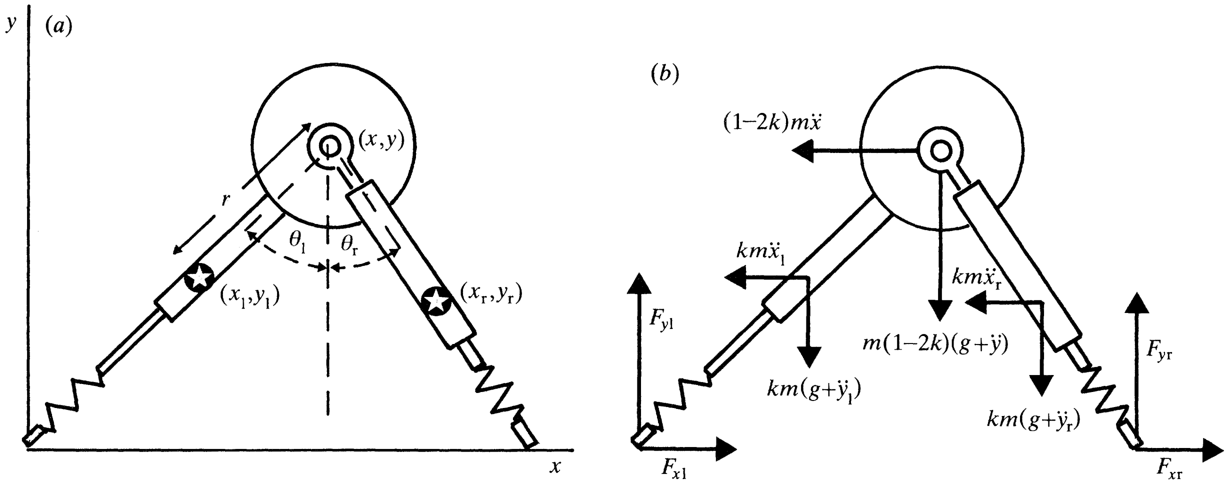

Figure 2. (*a*) The model described in the text. Stars mark the centres of mass of the legs. (b) A free body diagram showing the forces that act on the model.

(stance phase) and off (swing phase). The mid points of the stance phase of the left foot occur at times 0, T, $2T$ etc. and of the right foot at $0.5T$, $1.5T$, $2.5T$ etc. Thus T is the stride period. The duty factor is β, so the first stance phase of the left foot (for example) extends from time $-\beta T/2$ to $+\beta T/2$. At time t in this interval, the vertical component of the force that it exerts on the ground is given by

$$F_{y,\mathrm{left}} = \left\{\frac{3\pi mg}{4\beta(3+q)}\right\}\left\{\cos\left(\frac{\pi t}{\beta \mathrm{T}}\right) - q\cos\left(\frac{3\pi t}{\beta T}\right)\right\}. \qquad (1)$$

The term in the second set of braces is a truncated Fourier series, in which the factor q will be varied between -0.33 and 1.00 to give different patterns of force. The lack of sine terms and of even-numbered cosine terms ensures that the force is zero at the beginning and end of the stance phase and that the force pattern is symmetrical about zero time. For generality, the series would have to be continued with an infinite number of odd-numbered cosine terms, but it has been shown that the truncated series is capable of imitating reasonably well, the patterns of force that people exert when they walk and run (Alexander & Jayes 1980). The term in the first set of braces ensures that the mean vertical force exerted by the two feet over an integral number of strides is equal to the weight mg of the body.

Similarly, the right foot is on the ground from time $(1-\beta)T/2$ to $(1+\beta)T/2$ and during that time exerts a vertical component of force

$$F_{y,\mathrm{right}} = \left\{\frac{3\pi mg}{4\beta(3+q)}\right\}\left\{\cos\left[\frac{\pi}{\beta}\left(\frac{t}{T}-\frac{1}{2}\right)\right] - q\cos\left[\frac{3\pi}{\beta}\left(\frac{t}{T}-\frac{1}{2}\right)\right]\right\}. \qquad (2)$$

The model moves in Cartesian space taking strides of length λ. The coordinates of the hip joints are $(0,y_o)$ at time 0, (x,y) at time t and $(\lambda/2,y_o)$ at time $T/2$. The forces on the feet are always in line with the hip joints so, during the stance phases that we have been considering, the horizontal components of the forces on the two feet are given by

$$F_{x,\mathrm{left}} = x\,F_{y,\mathrm{left}}/y, \qquad (3)$$

$$F_{x,\mathrm{right}} = (x-\lambda/2)\,F_{y,\mathrm{right}}/y. \qquad (4)$$

This assumption is realistic, provided that the 'hip' joint of the model is not interpreted as being coincident with the human hip. Alexander & Jayes (1980) showed that equations (3) and (4) describe well the horizontal components of force exerted by human feet, provided that y is not interpreted as the height of the hip, but of a point in the body at 1.7 times the height of the hip. The forces on the feet of walking people remain more nearly in line with the anatomical hip than this may suggest, because the centre of pressure moves progressively from heel to toe during the stance phase (See Debrunner 1985).

Let the long axis of a leg (defined as the line through the hip joint and the leg's centre of mass) make an angle θ with the vertical. We will use subscripts to distinguish between the angle θ_l of the left leg, and θ_r of the right. While a foot is on the ground, its leg's angular velocity $\dot{\theta}$ is assumed constant and equal to $\bar{u}/y_o$, where $\bar{u}$ $(=\lambda/T)$ is the mean velocity of the biped over a complete stride. This conveniently simple assumption of constant angular velocity implies that the centre of mass of the leg remains very nearly but not precisely on the line from hip to foot. The foot leaves the ground at a time $\beta T/2$ after the mid point of the step, at which time the angle θ is $\beta\bar{u}T/2y_o$.

While the foot is off the ground, angle θ changes sinusoidally as in McGeer's (1990*a*) 'synthetic wheel' model of walking. For example, in the first swing phase of the left leg

$$\theta_l = A\sin\left[B\,(t-T/2)\right], \qquad (5)$$

where A and B are constants. This equation implies that the leg is vertical at the mid point of the swing phase, at time $T/2$. The values of A and B are fixed by the requirement of a smooth transition from stance phase to swing phase: at the instant when the foot leaves the ground, θ must be $\beta\bar{u}T/2y_o$ and $\dot{\theta}$ must be $\bar{u}/y_o$, as explained in the preceding paragraph. For any particular duty factor β, values of $2Ay_o/\bar{u}T$ and $BT/2$ can be calculated. Hence it can be shown that equation (5) implies that the range of angles through which the leg moves in the swing phase is 1.06 times the range for the stance phase in a walk with $\beta=0.6$, and 1.41 times the range for the stance phase in a run with $\beta=0.3$. This seems reasonably realistic.

Figure 2*b* is a free-body diagram of the biped. It is apparent that

$$F_{xl}+F_{xr} = (1-2k)\,m\ddot{x} + km\,(\ddot{x}_l+\ddot{x}_r). \qquad (6)$$

Note that the x-coordinate x_l of the centre of mass of the left leg is given by

$$x_l = x - r\sin\theta_l,$$

whence

$$\dot{x}_l = \dot{x} - r\,\dot{\theta}_l\cos\theta_l$$

$$\ddot{x}_l = \ddot{x} - r\,\ddot{\theta}_l\cos\theta_l + r\,\dot{\theta}_l^2\sin\theta_l,$$

and similarly for the right foot. Hence

$$F_{xl}+F_{xr} = m\ddot{x} - kmr\,(\ddot{\theta}_l\cos\theta_l + \ddot{\theta}_r\cos\theta_r - \dot{\theta}_l^2\sin\theta_l - \dot{\theta}_r^2\sin\theta_r),$$

$$\ddot{x} = (F_{xl}+F_{xr})/m + kr(\ddot{\theta}_l\cos\theta_l + \ddot{\theta}_r\cos\theta_r - \dot{\theta}_l^2\sin\theta_l - \dot{\theta}_r^2\sin\theta_r). \qquad (8)$$

Figure 2*b* also shows that

$$F_{yl}+F_{yr} = mg + (1-2k)\,m\ddot{y} + km\,(\ddot{y}_l+\ddot{y}_r), \qquad (9)$$

and

$$y_l = y - r\cos\theta_l,$$

$$\dot{y}_l = \dot{y} + r\,\dot{\theta}_l\sin\theta_l,$$

$$\ddot{y}_l = \ddot{y} + r\,\ddot{\theta}_l\sin\theta_l + \mathrm{r}\,\dot{\theta}_l^2\cos\theta_l, \qquad (10)$$

and similarly for the right foot. Hence

$$F_{yl}+F_{yr} = mg + m\ddot{y} + kmr\,(\ddot{\theta}_l\sin\theta_l + \ddot{\theta}_r\sin\theta_r + \dot{\theta}_l^2\cos\theta_l + \dot{\theta}_r^2\cos\theta_r),$$

$$\ddot{y} = (F_{yl}+F_{yr})/m - g - kr\,(\ddot{\theta}_l\sin\theta_l + \ddot{\theta}_r\sin\theta_r + \dot{\theta}_l^2\cos\theta_l + \dot{\theta}_r^2\cos\theta_r). \qquad (11)$$

The coordinates (x,y) of the hip joints at successive times t have been obtained by numerical integration of equations (8) and (11), using the information already given about the forces F (equations (1) to (4)) and leg angles θ.

The telescopic actuators that alter the lengths of the legs do work only while their feet are on the ground. During the first step, while the left foot is on the ground at (0,0) the length of the left leg is

$$l_l = (x^2 + y^2)^{\frac{1}{2}}. \qquad (12)$$

Similarly, while the right foot is on the ground at $(\lambda/2, 0)$

$$l_r = [(x - \lambda/2)^2 + y^2]^{\frac{1}{2}}. \qquad (13)$$

The resultant force on the left foot is

$$F_l = (F_{xl} + F_{yl})^{\frac{1}{2}}, \qquad (14)$$

always acting (as already specified) in line with the hip joint. In a time increment δt in which F_l increases by an increment δF_l, the spring in the left leg (compliance C) is compressed by $C \cdot \delta F_l$, storing strain energy $F_l C \cdot \delta F_l$. Thus the work done in this time increment by the telescopic actuator of the left leg is

$$\delta W_{l1} = F_l \ (\delta l_l + C \cdot \delta F_l), \qquad (15)$$

and similarly for the right leg. The numeral 1 in the subscript on the left hand side of equation (15) indicates that this work is done by the telescopic actuator: work done by the torque actuators will be distinguished by numerals 2.

To determine the work done by the torque actuators, we need to know the moments M at the hip joints. Take moments about the left hip in figure 2*b*, remembering that the resultant of F_{xl} and F_{yl} is aligned with the hip

$$M_l = kmr[\ddot{x}_l \cos \theta_l - (\ddot{y}_l + g) \sin \theta_l]. \qquad (16)$$

Substitute for $\ddot{x}_l$ and $\ddot{y}_l$, using equations (7) and (10), and obtain (after a little algebra)

$$M_l = kmr[\ddot{x} \cos \theta_l - (\ddot{y} + g) \sin \theta_l - r \ \ddot{\theta}_l]. \qquad (17)$$

A similar equation can be obtained for the right leg.

The work done by the left hip actuator in the time increment δt is

$$\delta W_{l2} = M_l \cdot \delta \theta_l. \qquad (18)$$

The model is frictionless and is travelling over level ground. It has the same velocity at corresponding stages of successive strides. Consequently, the net work done by the actuators, over a complete stride, is zero: positive work done by them at one stage of a stride is matched by negative work at another. (An actuator doing negative work behaves like a brake, degrading mechanical energy to heat.) As in Alexander (1980) we will estimate the energy cost of a stride by summing the increments of positive work only. To do this, we need actually consider only one quarter stride because the symmetry of the model's movements ensures that equal work is done in successive half strides, and positive work done in one quarter stride is matched by negative work done in the next. We will define the mechanical cost of transport H as the sum of the increments of positive work per unit weight of biped, done while travelling unit distance

$$H = (2/mg\lambda) \sum_{t=0}^{t=T/4} (|\delta W_{l1}| + |\delta W_{r1}| + |\delta W_{l2}| + |\delta W_{r2}|. \qquad (19)$$

The vertical lines || signify that the absolute values of the increments of work are to be summed, irrespective of sign. The sum is therefore equal to the sum of positive work increments over a half stride.

Symmetry requires that at time 0, when the hip is vertically over the supporting foot, the hip must be travelling horizontally. Calculations were performed on a microcomputer by starting the model at time 0, with the hips at $(0, y_o)$ and travelling horizontally. The movements of the model, and the work performed by the actuators, were determined for the first quarter stride by numerical integration, and the cost of transport was calculated by means of equation (19). The program tried different initial velocities dx/dt until one was found that gave a mean velocity u over the quarter stride, that was within 0.2% of the required value. The accuracy of the calculations was checked by increasing the number of integration steps by a factor of four, for a representative selection of combinations of speed, shape factor and duty factor. The increased number of steps altered the calculated cost of transport by more than 4% in only five out of 170 trials. Errors of this magnitude are trivial, for the purposes of this paper.

3. VALUES FOR PARAMETERS

Results will be presented in dimensionless form using y_o as the unit of length, m as the unit of mass and $(y_o/g)^{\frac{1}{2}}$ as the unit of time. There is therefore no need to specify particular values for leg length (represented by y_o) or body mass (m). However, values had to be chosen for the other anatomical parameters. They were selected to match approximately the proportions and properties of the human body.

Each leg of a human adult represents about 0.16 of total body mass (Winter 1990). Therefore k is taken to be 0.16. It can be calculated from data in Winter (1990) that the radius of gyration of the limb about the hip is about 0.52 of the height of the hip joint from the ground, so r/y_o is taken to be 0.52.

To select an appropriate elastic compliance C for the legs we will use the data of Ker *et al.* (1987), who calculated the strain energy stored in the Achilles tendon and foot of a 70 kg man running at a middle-distance speed. They estimated that 52 J strain energy was stored when the peak force of 1900 N acted on the foot. A force F on a linear spring of compliance C stores $F^2C/2$ strain energy, so we can estimate C as $2 \times 52/1900^2 = 2.9 \times 10^{-5}$ m N^{-1}. The dimensionless parameter that we require is Cmg/y_o. We have already noted that to make equations 3 and 4 model the horizontal components of force on human feet, we have to make y_o 1.7 times the length of the human leg, or about 1.6 metres. The gravitational acceleration g is 9.8 m s^{-2}. Thus Cmg/y_o is given the value $2.9 \times 10^{-5} \times 70 \times 9.8/1.6 = 0.012$. The appropriateness of this value will be considered further in the Discussion section.

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We have to choose for investigation appropriate ranges of shape factor q, duty factor β, speed $\bar{u}$ and stride length λ. If the vertical component of the force on the ground is to remain positive throughout the step, q is restricted to the range -0.33 to 1.00. We will therefore investigate only this range of shape factors. The duty factor β could in principle take any value between 0 and 1.

Speed will be represented by the dimensionless parameter $\bar{u}/(gy_o)^{\frac{1}{2}}$. With $g = 9.8$ m s^{-2} and $y_o = 1.6$ m (see above), this parameter is 0.25 times the numerical value of the speed in metres per second. Results will be presented for speed parameters ranging from 0.1 (corresponding to an extremely slow walk, at 0.4 m s^{-1}) to 1.2 (corresponding to a run at 5 m s^{-1}). Adult people commonly walk at speeds in the range 0.8 to 1.7 m s^{-1} (Bornstein & Bornstein 1976) and break into a run at speeds above 1.9 m s^{-1} (Thorstensson & Roberthson 1987).

Stride length λ increases with speed. Alexander & Maloiy (1984) give values for the relative stride length (λ/h), where h is the height of the hip joint from the ground. They show that for human walking and running, λ/h is approximately equal to 2.5 $[\bar{u}/(gh)^{\frac{1}{2}}]^{0.6}$. With $y_o = 1.7h$, as required to make equations (3) and (4) represent satisfactorily the horizontal forces exerted by human feet (see above), this makes λ/y_o approximately equal to 1.7 $[\bar{u}/(gy_o)^{\frac{1}{2}}]^{0.6}$. It will be convenient to define a stride factor S such that

$$\lambda/y_o = S\,[\bar{u}/(gy_o)^{\frac{1}{2}}]^{0.6}. \tag{20}$$

Many of the calculations will be made using the empirical value, $S = 1.7$, but results will also be presented for a range of stride factors so that optimum stride lengths can be predicted.

4. RESULTS

(*a*) *Stiff, massless legs*

Figure 3 shows the mechanical cost of transport when the legs have no mass $(k = 0)$ and no elastic compliance $(C = 0)$, making the model identical with the one investigated by Alexander (1980). Each of the graphs (*a*) to (*d*) shows results for a different speed. In each, lines of equal cost of transport are shown for the whole range of possible duty factors and shape factors. However, the effects of altering stride length are not shown: an appropriate stride length has been chosen for each speed by applying equation 20 with a stride factor of 1.7.

At a dimensionless speed of 0.1 (figure 3*a*), the model with stiff, massless legs uses least energy if it walks with high duty factors β and low shape factors q. The minimum cost of transport is obtained with $\beta = 0.9$, $q = -0.1$. Increasing the dimensionless speed to values not exceeding 1.0 reduces the optimum duty factor and increases the optimum shape factor. At a dimensionless speed of 0.3 (figure 3*b*) the optimum is close to $\beta = 0.7$, $q = 0.2$ and at 0.6 (figure 3*c*) it is approximately $\beta = 0.6$, $q = 0.6$. However, at dimensionless speeds above the critical value of 1.0 (figure 3*d*) the optimum gait is a run with zero duty factor.

As Alexander (1980) pointed out, this model successfully predicts the reduction in duty factor and increase in shape factor that are observed as people walk faster. It also predicts an abrupt change to running, at a critical speed. However, the predicted critical speed is much too high. For an adult human with $y_o = 1.6$ m (see above) it represents a speed of 4 m s^{-1}, but adults actually break into a run at only about 1.9 m s^{-1} (Thorstensson & Roberthson 1987). The

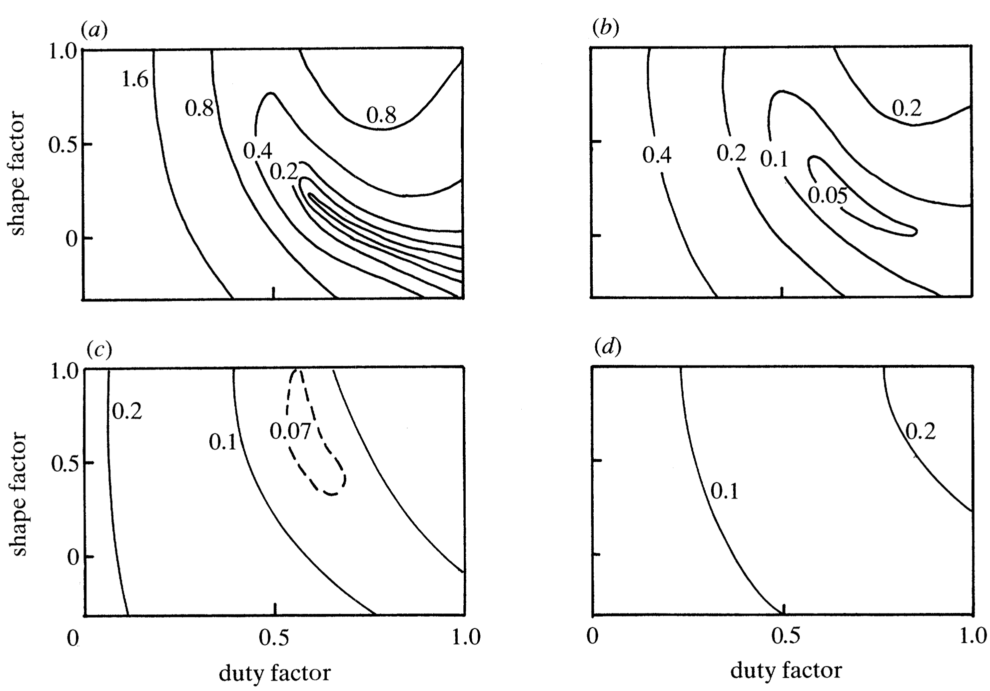

Figure 3. Mechanical costs of transport for the model with stiff, massless legs at dimensionless speeds of (*a*), 0.1; (*b*), 0.3; (*c*), 0.6 and (*d*), 1.2. Shape factor is plotted against duty factor, with contours showing costs of transport. Note that in this and subsequent figures, successive contours represent costs of transport differing by a factor of two (except in the case of contours shown as broken lines). The stride factor is 1.7 throughout.

next subsection will show that this prediction is changed when account is taken of the elastic compliance of the legs.

(*b*) *Compliant, massless legs*

Figure 4 shows costs of transport for the model when its legs are again given no mass ($k = 0$), but are given an elastic compliance based on that of human legs ($Cmg/y_0 = 0.012$: see above). The costs for high duty factors are little different from the corresponding costs for the stiff model, shown in figure 3, but each graph shows a deep minimum at low duty and shape factors. This represents a gait in which the telescopic actuator of each leg holds almost constant length, allowing the biped to bounce along on its springs like a rubber ball. A frictionless model like this with compliant, massless legs could travel with zero energy cost, at any chosen speed and stride length (Blickhan 1989; McMahon & Cheng 1990). The models presented in this paper give finite energy costs at the low-duty-factor minima, only because of a restriction that was adopted to avoid having an unmanageable number of variables: we restricted the range of admissible force patterns to those that could be represented by a two-term Fourier series (equation 1).

Figure 4 shows results only for duty factors of 0.1 or more, but if lower duty factors were included we would see costs approaching infinity as the duty factor approached zero. The reason is that at very low duty factors enormous forces act on the feet, causing very large compressions of the passive springs that have to be compensated by extension of the telescopic actuators. The strain energy stored in a leg after a footfall, at very low duty factors, is far more than the kinetic and potential energy loss, so most of it has to be supplied by the actuator. When the spring recoils it releases far more energy than is needed to increase the body's kinetic and potential energy, and the actuator must do negative work to degrade the excess to heat.

Thus the introduction of leg compliance has little effect on the model's energy costs at high duty factors, and profound effects at low ones. We will now note an effect at moderate duty factors. At a dimensionless speed of 0.3, the walking energy minimum occurs at a shape factor of about 0.2 for the stiff model (figure 3*b*) and 0.4 for the compliant one (figure 4*b*); and at a dimensionless speed of 0.6 the minimum occurs at a shape factor of 0.6 for the stiff model and 1.0 for the compliant (figure 3*c* and 4*c*). The introduction of compliance shifts this minimum to higher shape factors, especially at faster walking speeds.

Figures 4*a*,*b*, and *c* show two minima each, one at a low duty factor (running) and the other at a high duty factor (walking). We have noted that optimal running would require no positive or negative work from the actuators, if the restriction on the force patterns imposed by equation (1) were relaxed. There is no reason to believe that this or any other reasonable change to the model would reduce the work requirement at the walking minimum to zero. This suggests that if work minimization were the sole criterion, compliant-legged bipeds should always run, no matter how slowly they were travelling. Why then do humans and animals walk at low speeds? We will return to this problem in the Discussion section.

(*c*) *Legs with mass and compliance*

Figure 5 shows results for a biped whose legs have

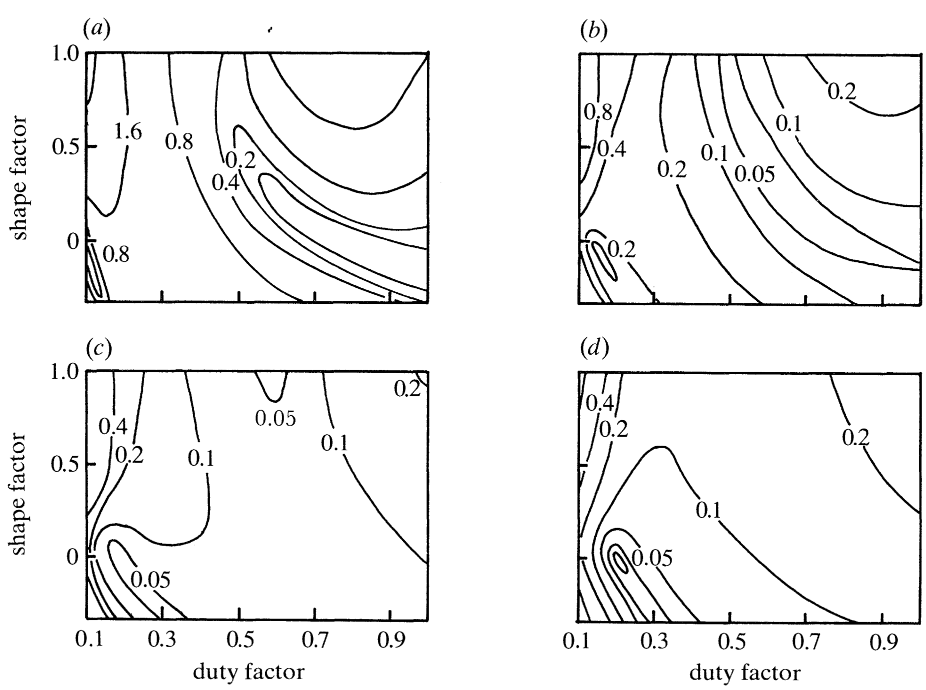

Figure 4. Mechanical costs of transport for the model with compliant, massless legs at dimensionless speeds of (*a*), 0.1; (*b*), 0.3; (*c*), 0.6; and (*d*), 1.2. Shape factor is plotted against duty factor, with contours showing costs of transport. The stride factor is 1.7 throughout.

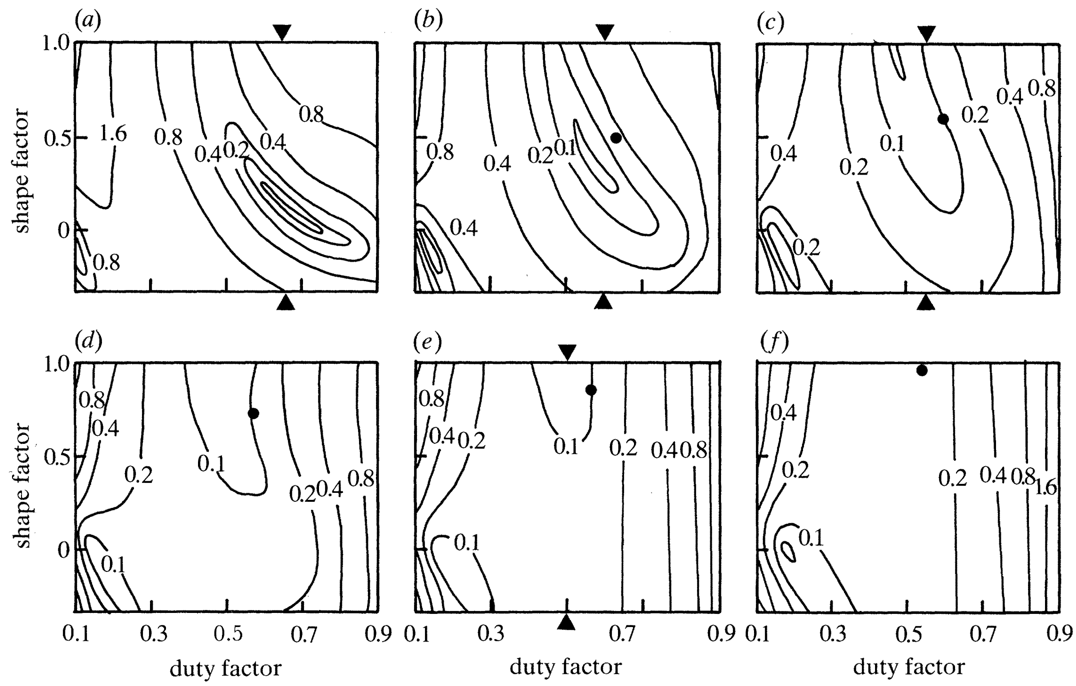

Figure 5. Mechanical costs of transport for the model whose legs have mass and compliance, at dimensionless speeds of (*a*), 0.1; (*b*), 0.2; (*c*), 0.3; (*d*), 0.4; (*e*), 0.5; and (*f*), 0.6. Shape factor is plotted against duty factor, with contours showing costs of transport. The stride factor is 1.7 throughout. Arrows on some of the graphs indicate the duty factors to which the graphs for the same speeds in figure 6 apply. Points are calculated from the empirical equations of Alexander & Jayes (1980).

mass as well as elastic compliance. Note that some of the speeds chosen for illustration are different from those that appear in Figures 3 and 4.

Because the legs now have mass, the hip actuators may have to do work to give them kinetic energy at the start of each forward or backward swing, and negative work to stop them swinging at the end. This would not be the case if the duration of the swing phase were long enough for the leg to swing passively forward as a pendulum, but the swing phase is too short for this in the range of speeds considered in this paper, unless stride factors longer than the empirical value of 1.7 are used.

The work required of the hip actuators is low at low duty factors and higher at high ones, which allow less time for the forward swing of the leg. It approaches infinity as the duty factor approaches 1.0. For this reason, the walking energy minima shown in figure 5 are shifted to lower duty factors, in comparison with the minima for the same speeds in figures 3 and 4. The effect is most marked at the lowest speed: the walking energy minimum occurs at a duty factor of about 0.9 in figure 4*a*, but at about 0.65 in figure 5*a*.

This model, unlike the one with massless legs, cannot run without energy cost. However, the running energy minima in figure 5 are still the global minima because leg mass adds less to the work required for locomotion at low duty factors, than at high ones.

So far we have set stride lengths to the values calculated for each speed from equation (20), using the empirical value of 1.7 for the stride factor *S*. Now we will investigate the effect of varying stride length. Figure 6 shows for several speeds how work requirements vary with shape factor and stride factor, if duty factor is held constant. The duty factors have been chosen to correspond approximately with the walking energy minima shown in figure 5.

Each of the graphs in figure 6 shows that the cost of transport has a minimum value, at a particular combination of shape factor and stride factor. In every case this stride factor lies between 1.5 and 2.0, so is close to the empirical value of 1.7.

The set of possible gaits permitted by the model (for any particular speed) can be represented as a three-dimensional space, in which the dimensions are duty factor, shape factor and stride factor. Figures 5 and 6 show mutually perpendicular sections through this space. It seems that the walking gaits which have minimum cost of transport lie close to the intersections of the shape factor – duty factor graphs (figure 5) and the corresponding shape factor – stride factor graphs (figure 6).

Although the model indicates an optimum stride length for walking at any given speed, it does not give optima for running. It is possible to find a running gait that requires no work from the telescopic actuators, for any combination of speed and stride length. The longer the stride, the fewer strides are needed to cover a given distance and the less work is required of the hip actuators. Thus the model seems to tell us that a runner should take the longest possible strides. Runners actually increase stride length, as they increase speed (Högberg 1952). The model of McMahon & Cheng (1990) gave an optimum stride length for each speed of running, only because they

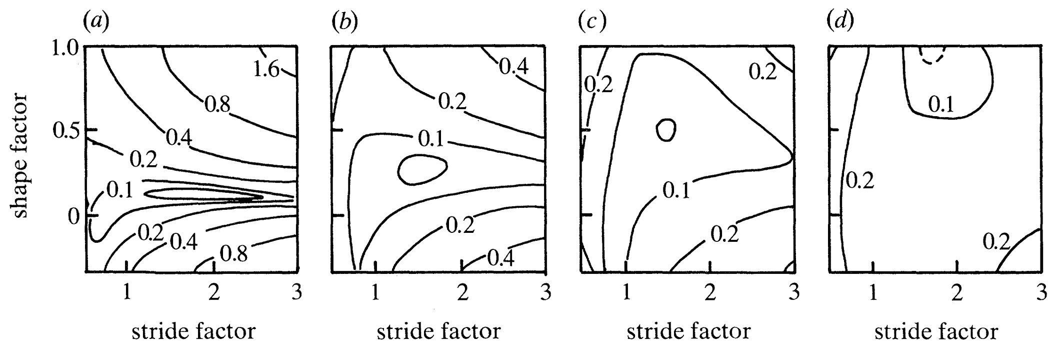

Figure 6. The effect of varying stride length on the mechanical cost of transport of the model whose legs have mass and compliance. The dimensionless speeds are (*a*), 0.1; (*b*), 0.2; (*c*), 0.3; and (*d*), 0.5. Shape factor is plotted against stride factor, with contours showing cost of transport. Duty factors are as indicated by arrows on the graphs for the same speeds in figure 5.

constrained their model in a way that we have not. They specified that the vertical component of the take-off velocity should be the same for all forward speeds, so that the runner should always rise and fall through the same vertical range during the aerial phases of its strides. With this constraint, their model was capable of reproducing the observed relationship between human and animal stride lengths, and running speed.

5. DISCUSSION

(*a*) *Limitations of the model*

The model has been kept extremely simple. For some purposes, we would require a model that imitates the human body much more closely, but extreme simplicity seems a merit for our present purpose, to explain why we change our gaits as we do, as we increase speed.

Several features of the model seem rather unsatisfactory, but it is not obvious how they could be remedied without making it more complex.

First, the model's 'hip' does not correspond to the anatomical hip. To make equations (3) and (4) model satisfactorily the forces exerted by human feet, we had to regard the model's hip height y as 1.7 times the hip height h of an equivalent human. Consequently, the period with which the model's leg would swing as a pendulum is $\sqrt{1.7} = 1.3$ times the pendulum period of the leg of an equivalent human. This could not have been remedied by moving the centre of mass higher up the model's leg: if this had been done, the fluctuations of the kinetic energy of the swinging leg would have been unrealistically small.

Secondly, the model has no knee joint. Mochon & McMahon (1980) showed how the time taken for a leg to swing forward passively is affected by the presence of a moveable knee joint.

Thirdly, the elastic compliance of the leg is assumed to be constant. When the leg is straight, as in the stance phase of walking, the Achilles tendon and the arch of the foot are probably the most important contributors to leg compliance, as assumed here. In the stance phase of running, however, the knee is somewhat bent and substantial force is required in the quadriceps muscles. The quadriceps tendon may then add substantially to the overall compliance of the leg.

Finally, it has been assumed that the quantity to be minimized is the work performed by the muscles. C. R. Taylor and his colleagues have argued forcefully (for example, in Kram & Taylor (1990)) that the forces developed by muscles may be more important than the work that they do, in determining the metabolic energy cost of running. It seems clear that work performance and force development both affect the metabolic cost of locomotion, which seems to be the cost we should wish to minimize. Unfortunately, our understanding of muscle function in locomotion is not yet good enough to enable us to predict metabolic costs from mechanical performance (Alexander 1991).

Despite these limitations, the model seems capable of improving our understanding of walking and running.

(*b*) *Comparison of predictions with observed gaits*

Alexander & Jayes (1980) made force plate records of human walking and derived equations relating duty factor and shape factor to speed. The points in figure 5 show the values predicted by their equations, for each speed. In calculating these values from the equation, it was of course necessary to remember that the model's hip height y_0 (used in the definition of dimensionless speed) is 1.7 times the hip height of an equivalent human. No point is shown in figure 5*a* because it represents a lower speed than any of the records on which the equations are based.

The empirical points in figure 5 show duty factor decreasing and shape factor increasing, as speed increases. The walking work minima predicted by the model (and shown by contours) are affected by speed in the same way. The empirical points are all quite near the predicted minima, but do not coincide with them. The predicted duty factors tend to be lower than the observed ones.

For each of the graphs in figure 5, the stride length was given a value appropriate to the speed calculated from the human stride lengths given by Alexander & Maloiy (1984). Stride length is varied in each of the graphs in figure 6, and duty factor is held constant.

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These duty factors were chosen to be close to the values at the walking work minima predicted by the model, but are also close to the values given for each speed by Alexander & Jayes' (1980) empirical equations. Figure 6 shows work minima at stride factors between 1.5 and 2.0, close to the empirical value of 1.7. Thus stride length at the walking work minima increases with speed, much as observed stride lengths increase with speed.

The model predicts work requirements for all possible combinations of duty factor, shape factor and stride length. We have seen that the values of these quantities used by people when they walk are close to those required to minimize the work required of the model's actuators if the model is constrained to walk (i.e. to use duty factors of 0.5 or more). However, it has been shown that for every speed a running gait can be found, which requires less work. To minimize the work required of its actuators the model should always run, even at low speeds. Why do people walk?

Measurements of the oxygen consumption of adult humans have shown that at speeds below about 2.2 m s^{-1}, walking requires less metabolic energy than running, but at higher speeds the converse is true (Margaria 1976). People change from walking to running at about this speed, which corresponds to a dimensionless speed for the model of 0.55. Why does the model fail to predict that walking will be more economical than running at low speeds? There are at least three possible reasons.

First, the model predicts very low duty factors (therefore, very large forces) for the running gaits that minimize work requirements. These duty factors would be higher if account were taken of the likely difference of leg compliance between walking and running, as discussed above. Even in that case, however, predicted optimum duty factor would still increase with increasing speed. Suppose that duty factors below some critical value are impossible, because they would imply forces that the leg could not withstand. In that case, the low costs of transport that the model predicts for optimal running at low speeds could not be attained, and the optimal walking gait might be more economical than any possible running gait.

Secondly, running with low duty factors requires muscles that can develop force and relax very rapidly, so fast muscles might be needed; but fast muscles are less economical than slow ones (Heglund & Cavagna (1987)).

Finally, in formulating the model we made the unrealistic assumption of perfect elasticity. The structures that serve as springs in human legs do not return, in their elastic recoil, all the work that was done to deform them. Instead, some of the energy is dissipated as heat. The energy dissipation is about 7% for tendon (Bennett *et al.* 1986) and may be as much as 22% for the arch of the foot (Ker *et al.* 1987). When the model with perfect springs runs, very little work is required of the actuators, but the springs store and return very large quantities of strain energy. If the springs were imperfect, dissipating some of this energy, the actuators might have to do substantial work to replace it and the advantage might shift from running to walking.

To assess the possible effects of imperfect elasticity, a few calculations have been performed in which it has been assumed that whenever strain energy is stored in a leg, a fraction is dissipated and has to be replaced by work done by muscles. If the strain energy increases by δE, energy $a \cdot \delta E$ is lost, where a is the fractional energy dissipation. Thus the mechanical cost of transport is given by a modified form of equation (19).

$$H = (2/\mathrm{mg}\lambda) \sum_{t=0}^{t=T/4} (|\delta W_{l1}| + |\delta W_{r1}| + |\delta W_{l2}| + |\delta W_{r2}| + a|\delta E_l| + a|\delta E_r|). \quad (21)$$

Adult humans break into a run at about 2 m s^{-1}, corresponding to a dimensionless speed for the model of 0.5. Accordingly, a few calculations were performed using equation (21) to discover how large a fractional energy dissipation would be needed to make optimal walking more economical than optimal running, at a dimensionless speed of 0.5. At this speed, with the stride factors fixed at 1.7, the walking work minimum was found to occur at a duty factor of 0.50 and a shape factor of 1.00; and the running minimum at a duty factor of 0.17 and a shape factor of -0.02 (figure 5*e*). At these minima, the mechanical costs of transport for the frictionless model are 0.074 for walking and 0.027 for running: the latter is considerably the more economical. As the fractional energy dissipation a is increased, the cost of running increases faster than the cost of walking, overtaking it when the energy dissipation is 0.31. (Both costs are then 0.097.) Thus the observed speed of transition from walking to running is not predicted by the modified model unless the energy dissipation is given this value, which is very much higher than the measured values (noted above) for tendon and for the arch of the foot.

For a slow running speed (figure 5*f*) the model predicts an optimum shape factor of zero and an optimum duty factor of 0.2. The shape factor is reasonably close to the mean value of -0.1 reported for human runners by Alexander & Jayes (1980). The duty factor, however, is much lower than the values of 0.35–0.40 used by humans running slowly (Alexander & Jayes 1980). Even in faster running, the duty factor does not fall below about 0.27 (Högberg 1952). To double the optimum duty factor to make it match the observed value for slow running, it would be necessary to increase the compliance of the legs by a factor of four. We have noted that the compliance used in our calculations is probably too low for a leg with its knee bent in the support phase of running, but it seems unlikely to be wrong by a factor of four.

Thus the model seems to explain many aspects of human walking and running, but leaves some questions unanswered. It does not make it clear why walking is preferred to running at low speeds, or why runners use such high duty factors. It remains uncertain whether this is because the gait that minimizes work may not be the one that minimizes metabolic costs, or for some other reason.

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